Fair Clustering Tutorial

Outline

- **Clustering paradigms** + short introduction to **algorithmic fairness**
- **Demographic** (group) fairness in clustering
- **Individual** fairness in clustering
- Algorithmic Aspect: The **two-stage approach** for solving fair clustering
- **Overlooked issues** in fair clustering.

Clustering Paradigms

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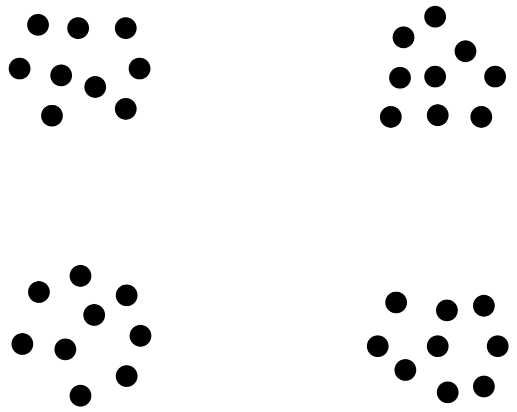

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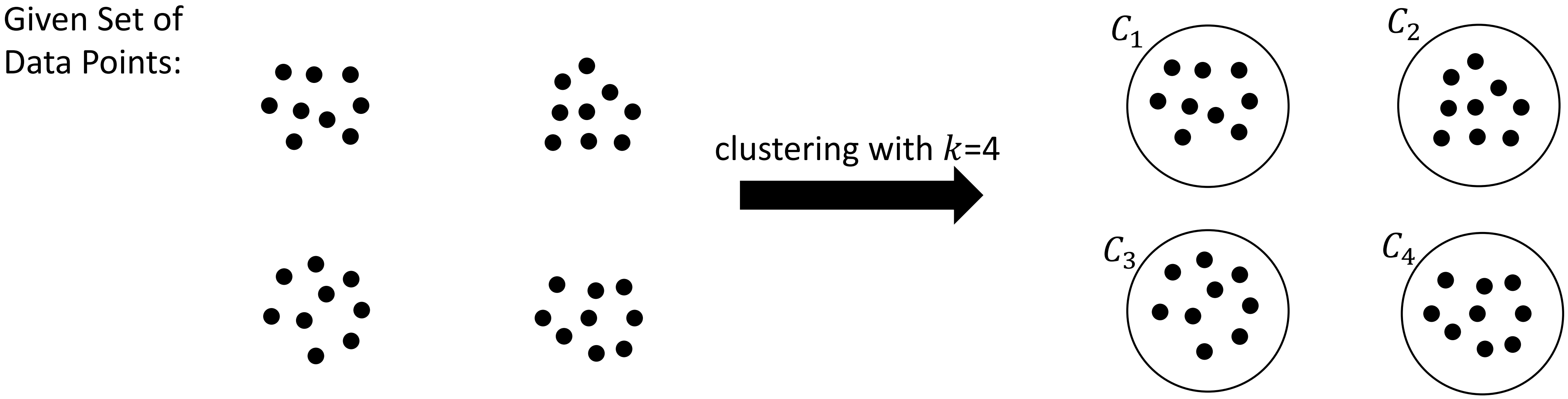

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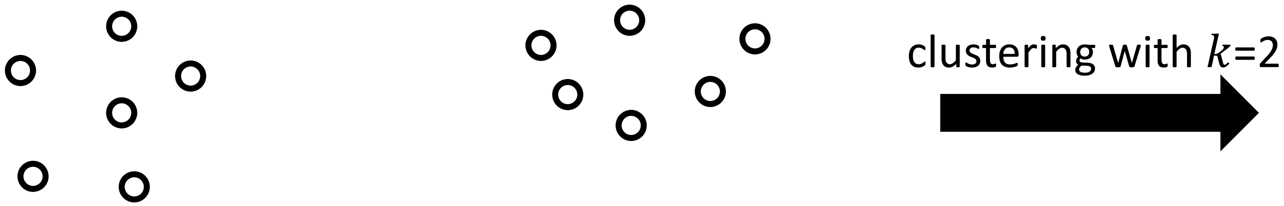

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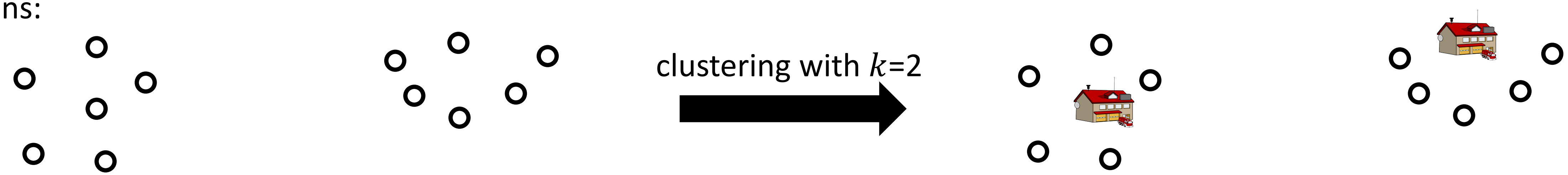

Clustering Paradigms

Clustering Paradigms

- Center-Based Clustering
- Spectral Clustering
- Correlation Clustering
- Hierarchical Clustering

Center-Based Clustering

- the cluster is decided by choosing k ***centers*** → each point is then assigned to ***its closest center***

- Includes k-means, k-median, and k-center clustering

Formalizing Center-Based Clustering

- *Input:*
  - Set of points: $\mathcal{C}$
  - Distance between points: $\forall i, j \in \mathcal{C}$ we have $d(i, j)$ (which is a ***Metric***)
  - Number of Clusters: k

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- *Output:*
  - Set of centers: S ($|S| \leq k$)
  - Assignment Function: $\varphi: \mathcal{C} \rightarrow S$ (assigning points to centers)

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- In General: $\min_{S,\varphi} \left\| \left[d\big(1, \varphi(1)\big),\ d\big(2, \varphi(2)\big), \ldots,\ d\big(n, \varphi(n)\big) \right] \right\|_p$

$p = \infty \rightarrow k$-center, $p = 1 \rightarrow k$-median, $p = 2 \rightarrow k$-means

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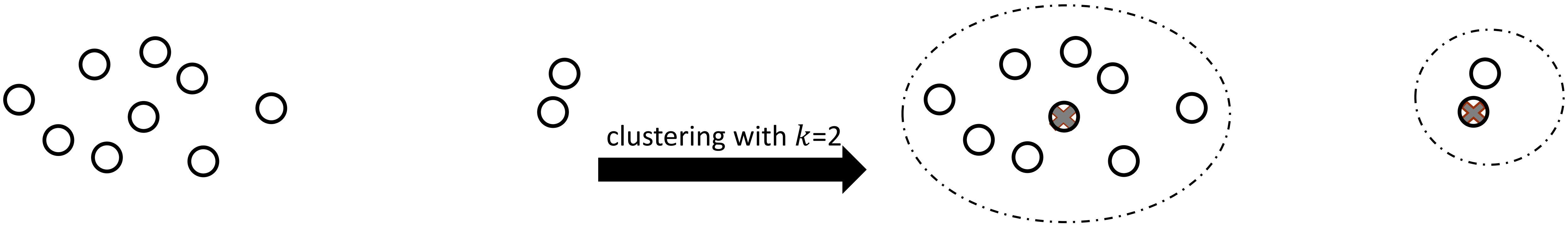

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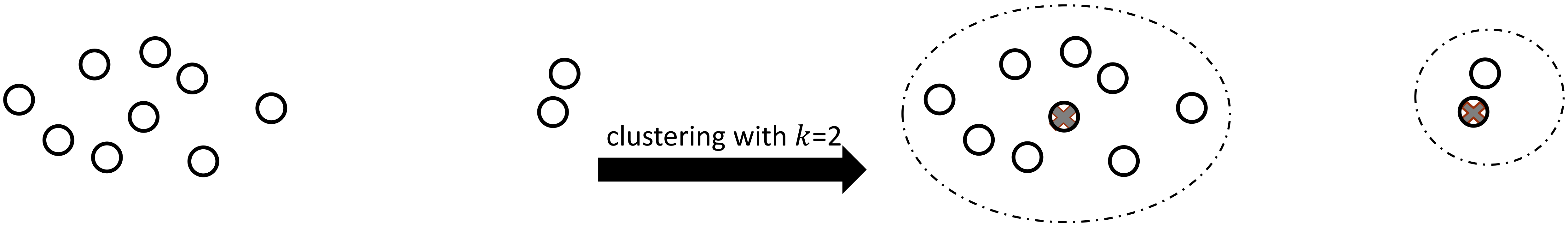

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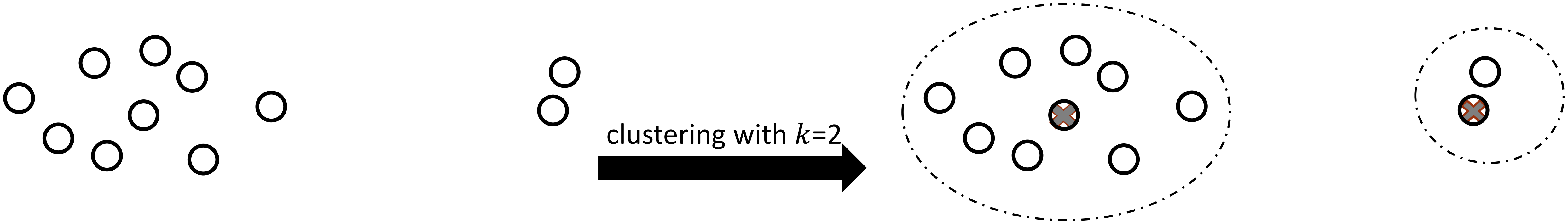

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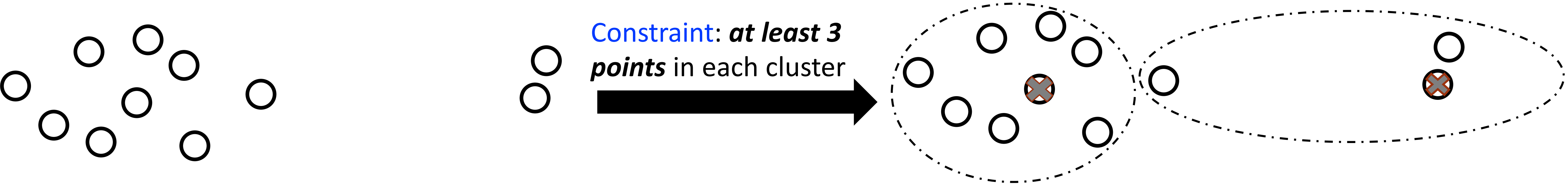

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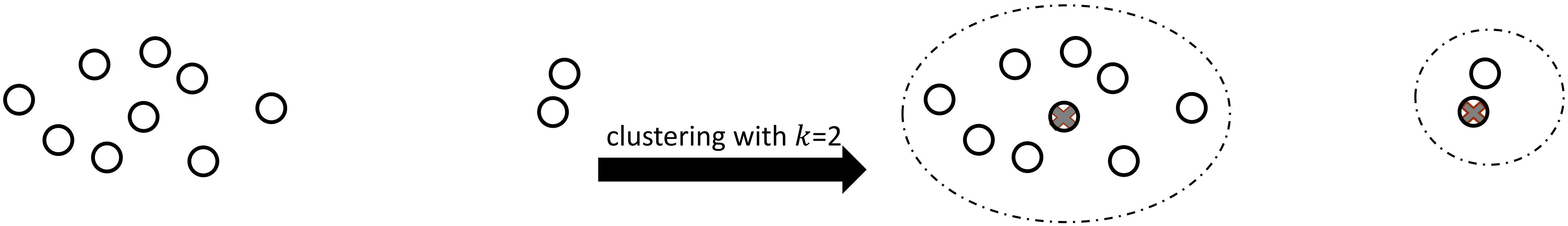

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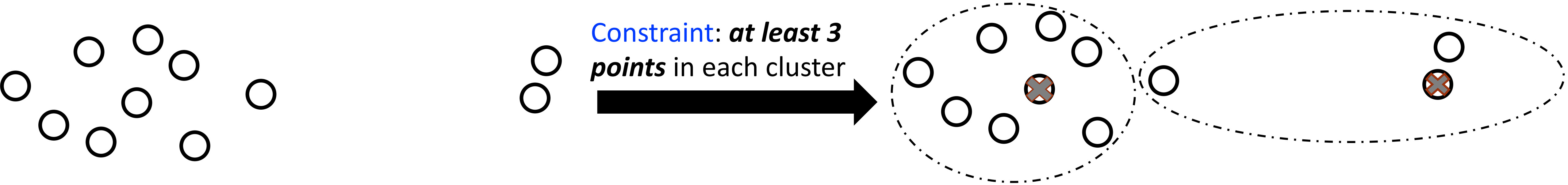
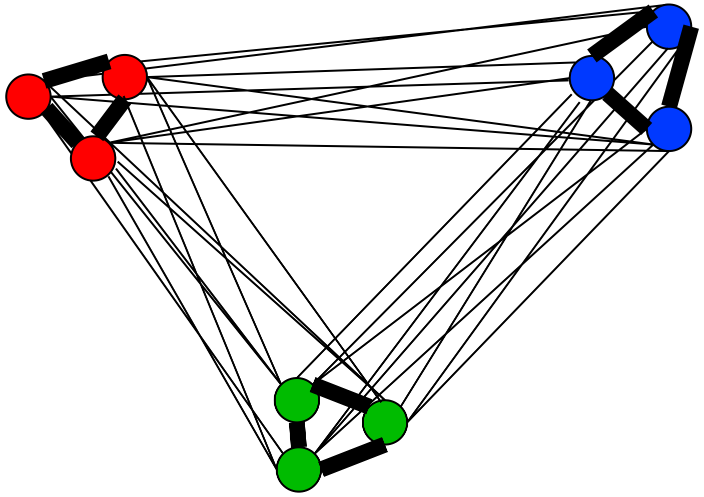


- Most Fair Clustering = Clustering **subject to *fairness constraints*** → Assigning points to centers is not trivial

Spectral Clustering

- Points are vertices in a graph $G = (V, E)$
- $\forall i, j: w_{ij} \geq 0$, w_{ij} is the similarity between i and j



Spectral Clustering

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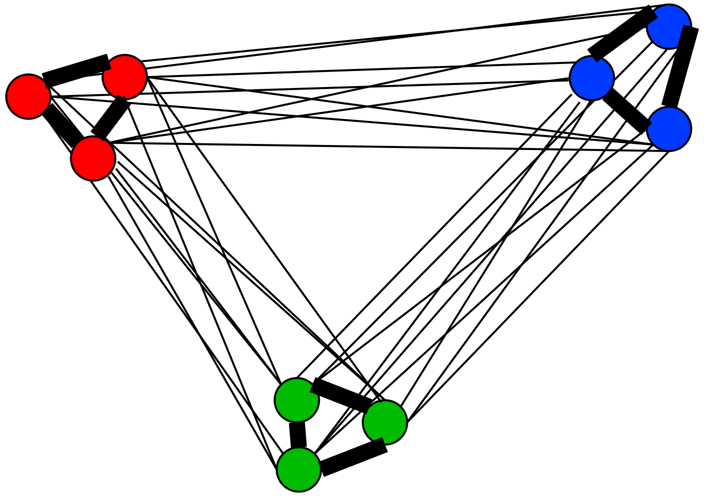
- Cluster (partition) the graph so that the edges *between points in **different** clusters* have low weight
- From the above, we have a graph cut problem



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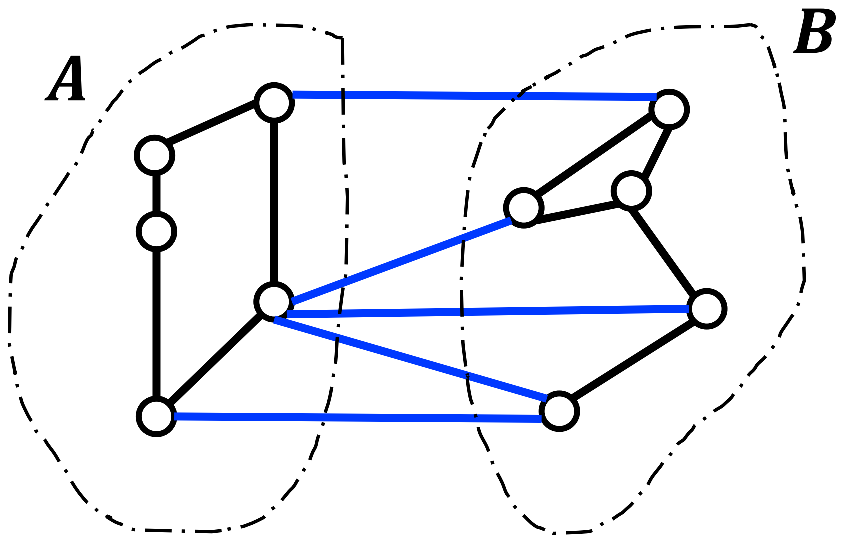
- $A, B \subset V$, $cut(A, B) = \sum_{i \in A, j \in B} w_{ij}$



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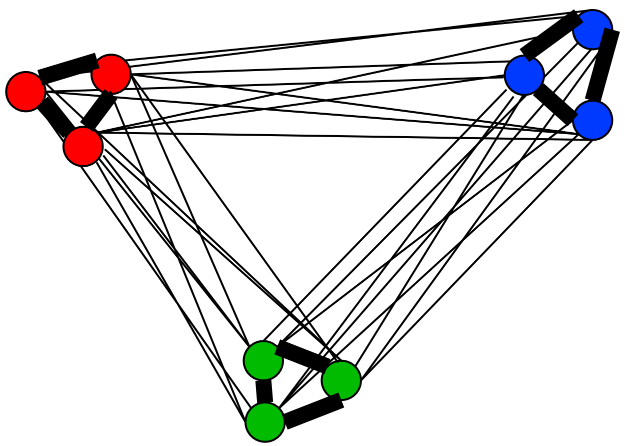

- Objective Functions:
  - Given graph $G = (V, E)$ and number of clusters k → Partition V into $C_1, \ldots, C_k$

$$\min \sum_{i=1}^{k} \frac{cut(C_i, V \setminus C_i)}{size(C_i)} \qquad , size(C_i) = ??$$

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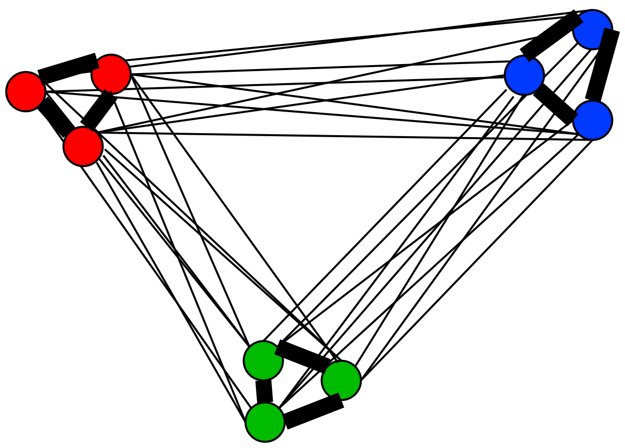

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(1) $size(C_i) = |C_i|$ → $\min \boldsymbol{RatioCut}(C_1, \dots, C_k) = \sum_{i=1}^{k} \frac{cut(C_i, V \setminus C_i)}{|C_i|}$

(2) $size(C_i) = \sum_{j \in C_i} d_j$ where $d_j = \sum_j' w_{jj}'$ → $\min \boldsymbol{NormalizedCut}(C_1, \dots, C_k) = \sum_{i=1}^{k} \frac{cut(C_i, V \setminus C_i)}{\sum_{j \in C_i} d_j}$

Correlation Clustering

- Points are vertices in a graph $G = (V, E)$
- $\forall i, j: w_{ij}^{+} \geq 0, w_{ij}^{-} \geq 0,$

  -w_{ij}^{+} is the degree to which i and j are **similar**

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  -*large* w_{ij}^{+} *edges* within a cluster

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- Cluster (partition) the graph so that you get

  -*low weight edges* between *different clusters*
  -*high weight edges* within a cluster

- Objective Function:

  Given graph $G = (V, E)$ → Partition V into $C_1, \ldots, C_k$

  $$\max \sum_{i,j: same\ cluster} w_{ij}^{+} + \sum_{i,j:\ different\ clusters} w_{ij}^{-}$$

Number of clusters k does NOT need to be given in correlation clustering

Hierarchical Clustering

- Like correlation clustering, you don't need to set k (the number of clusters)

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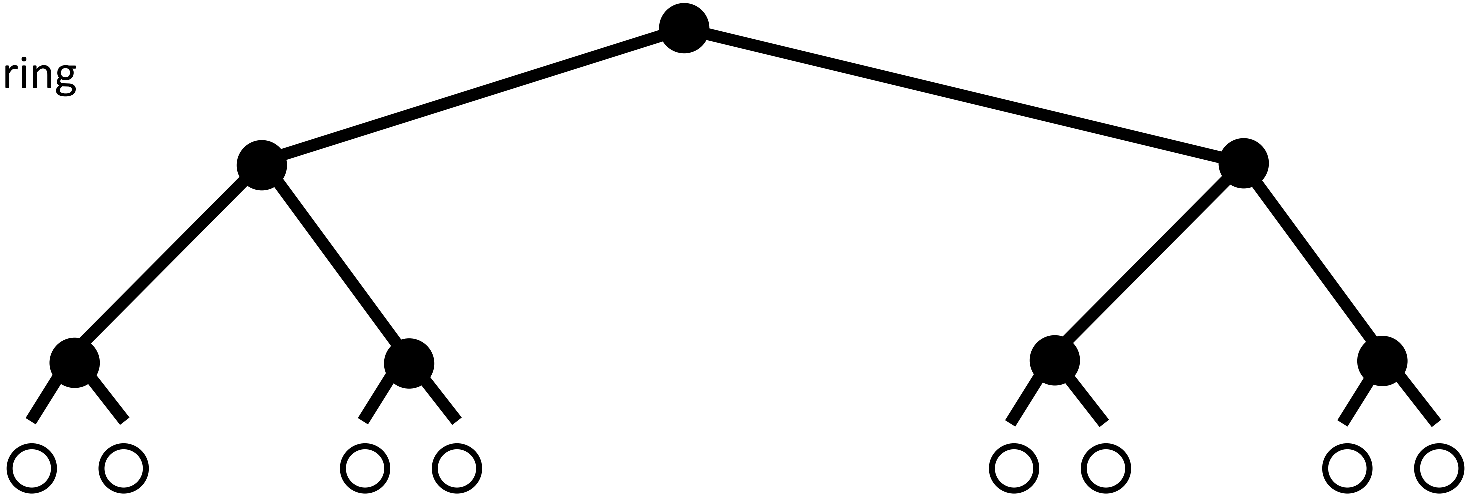
Hierarchical Clustering Output:



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- **agglomerative clustering** (common traditional method):

  -*bottom-up approach*: group similar points together forming a cluster, then group similar clusters and so on.

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- [Dasgupta, 2016] defines a cost function for hierarchical clustering:

  -Given $G = (V, E)$ with w_{ij} specifiying the similarity between i *and* j

  Objective Function:

  $\min \sum_{i,j} w_{ij} \times (\#\text{of descendents of lowest commen ancestor of } i \text{ and } j)$

  -the objective places higher penalty when separating points higher in the tree

- Further objectives for hierarchical clustering were also introduced [Moseley & Wang, 2017 ; Cohen-Addad et al, 2018].

Clustering Paradigms

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min objective value is OPT
→ solve for $\widehat{OPT}$,

$$\widehat{OPT} \leq \alpha OPT$$

α is the approximation ratio (clearly $\alpha > 1$)

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  Examples:

  -*loan approval* [Leo et al, 2018].

  -*recidivism prediction*[Goel et al, 2018]

  -*health care*: receiving limited resources [Wilder et al, 2018],

  kidney exchange [Dickerson et al, 2018]

Algorithmic Fairness

- Much of decision making is done at least partly using algorithms.
- Documented cases of algorithmic bias [O’Neil 2016; Kearns & Roth 2019].
- Substantial progress and interest in algorithmic fairness.

Some Considerations for Fairness in Clustering

- For a point i, its **distance from the center** $d(i, \phi(i))$:

  -closer to the center →better represented by the center (**ML**)

  -closer to the center →less travel distance to the facility (**OR**)

  → points want to be closer the center

- How does a fairness guarantee over the distance look like? How can we achieve that?

  - what if a group is consistently away from the center?

Some Considerations for Fairness in Clustering

- Clustering partitions the set of points $\mathcal{C}$ into clusters $C_1, C_2, \ldots, C_k$

  -different clusters will be processed differently, enjoy different outcomes, etc

  -suppose one demographic is under-represented in a cluster or over-represented in another.

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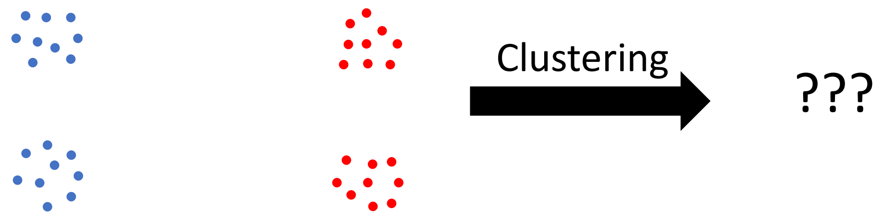

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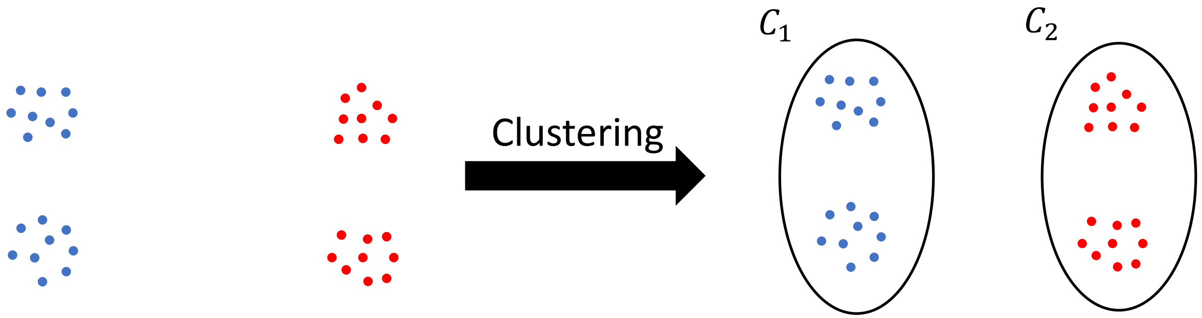

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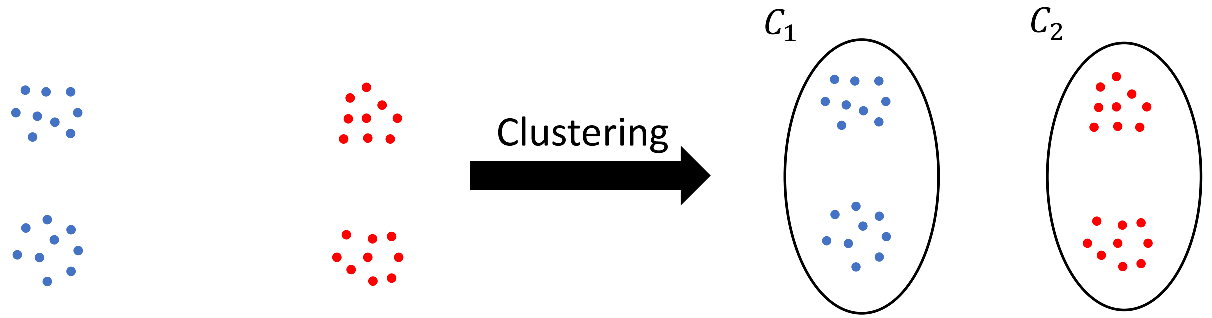

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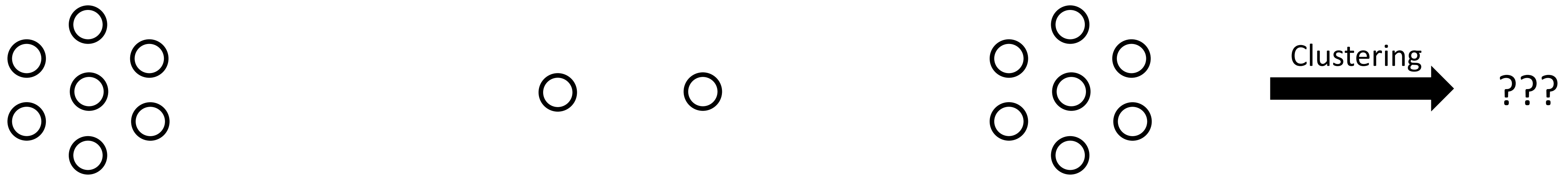

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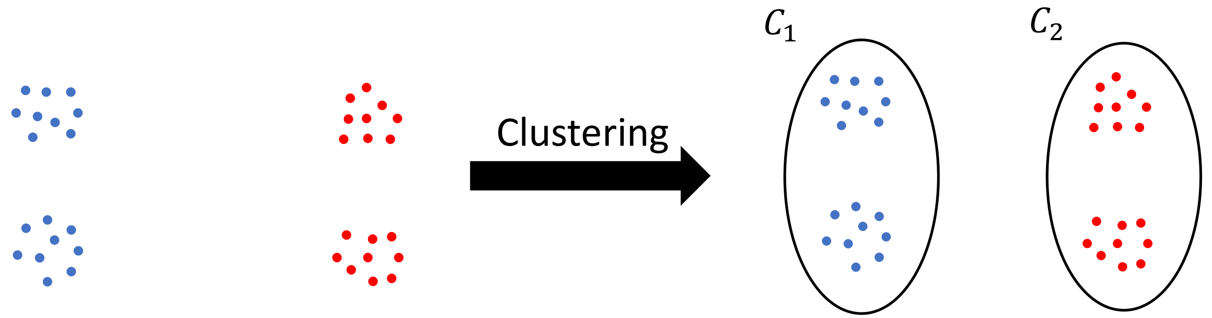

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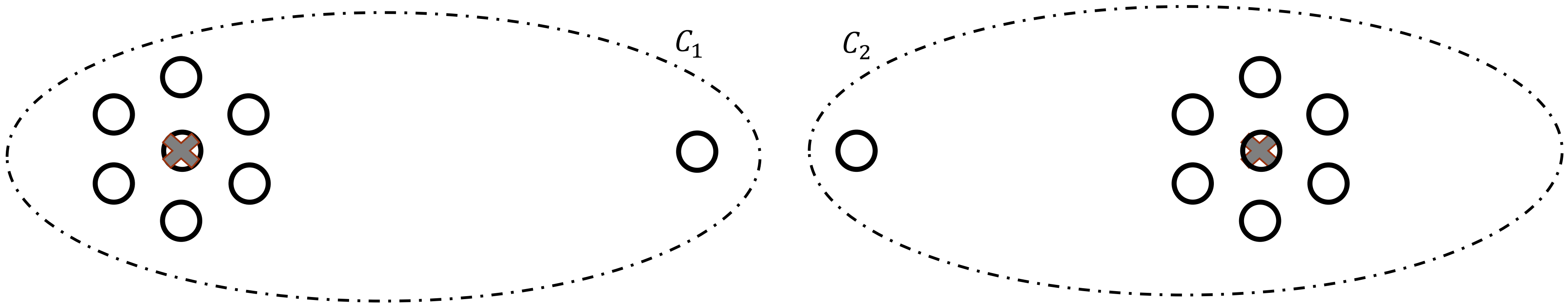

THANK YOU!