Demographic Fairness: Balance

Inspiration

Disparate Impact [FFMSV 15]

- *Griggs vs Duke Power Co.:* used non-racial features (notably, employee testing) as a proxy for race in order to discriminate against black employees in promotion
  - Duke Power Co. lost in the Supreme court, and this was deemed unlawful
  - This ruling and general philosophy helped promote affirmative action and anti-discrimination laws
  - Why is this bad:

    It had a "disproportionate and adverse impact on certain individuals."

    In other words, disparate impact.
- Applied to ML: Ensure that the impact of a system across protected groups is proportionate.
- Applied to Clustering:
  - The impact on a group is measured by how many individuals of that group are in each cluster.
  - Thus, we must ensure that the number of individuals from each group in each cluster is proportional to group size

Demographic Fairness - Balance

Recall: we are given points $\mathcal{C}$ in a metric space. We pick centers $S \subseteq \mathcal{C}$ and create a map $\varphi: \mathcal{C} \to S$. We also represent the clustering as a partitioning of points *S.*

Assume the points in $\mathcal{C}$ are given colors *red* or *blue*, representing protected classes.

For a cluster C :

$$balance(C) = \min\left(\frac{\#red(C)}{\#blue(C)}, \frac{\#blue(C)}{\#red(C)}\right)$$

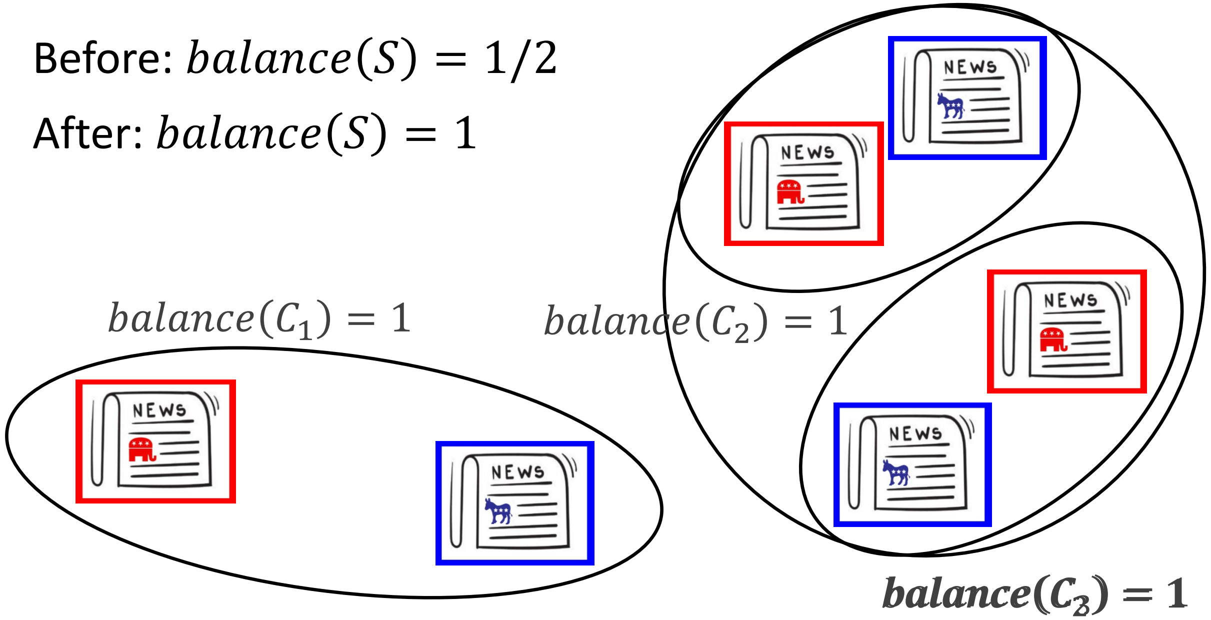
For a clustering S:

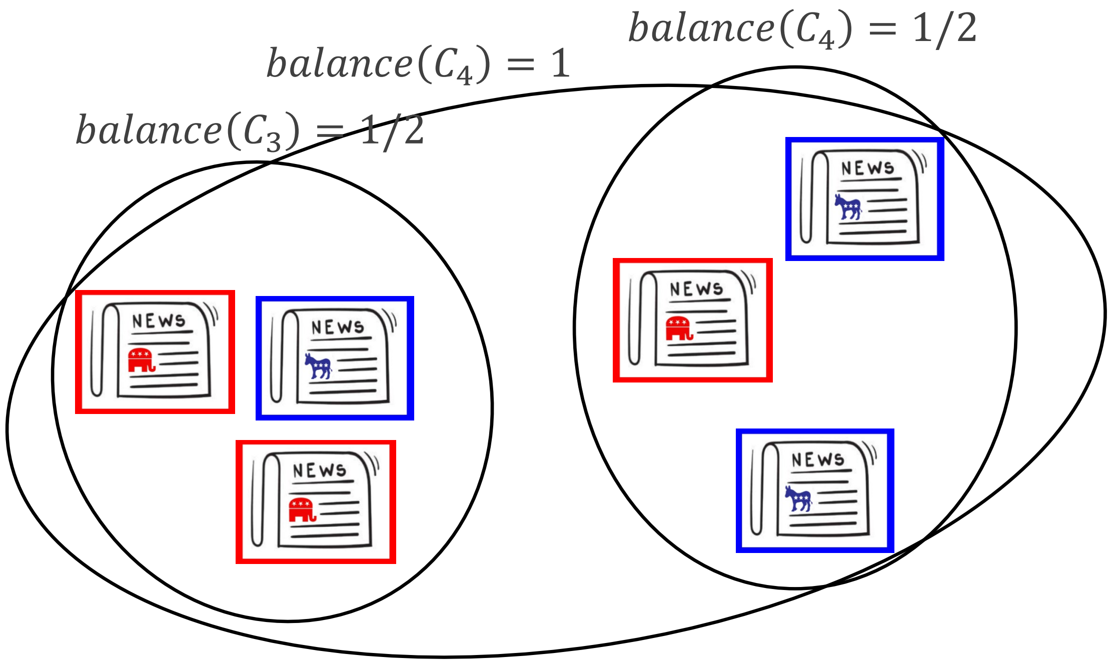
$$balance(S) = \min_{C \in S} balance(C)$$

We want balance to be high (close to 1).

Before: $balance(S) = 1/2$

After: $balance(S) = 1$

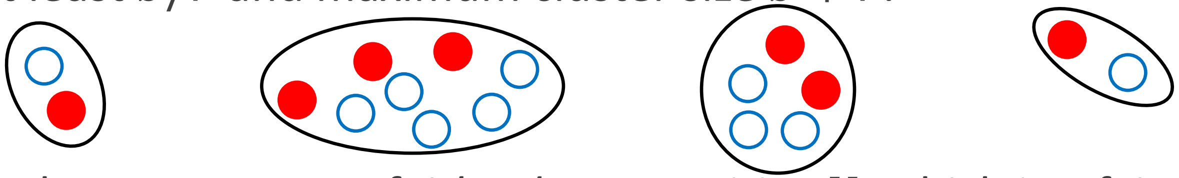




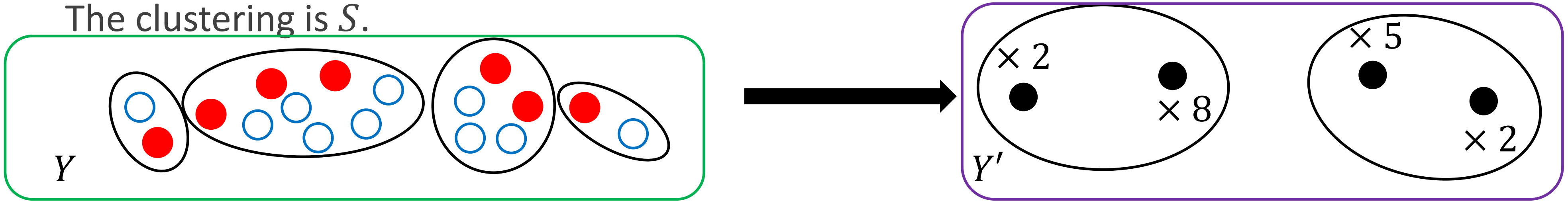
Results for Balance [CKLV 18]

Lemma: Let $balance(\mathcal{C}) \geq b/r$ for minimum integral b and r. Then we can find a clustering S with balance at least b/r and maximum cluster size $b + r$.

- *7 red, 10 blue*
- $balance(\mathcal{C}) \geq 3/5$



Method: Using the previous lemma, create a *fairlet decomposition* Y, which is a fair clustering with small (but possibly too many) clusters. Run a vanilla clustering algorithm on the fairlets centers as points duplicated to equal the size of the *fairlet*, call this set Y'. The clustering is S.



Structural result: for *k*-median and *k*-center, let the objective value be ψ:

$$\psi(\mathcal{C}, S) = \boxed{\psi(\mathcal{C}, Y)} + \boxed{\psi(Y', S)}$$

Further Results for Balance [CKLV 18]

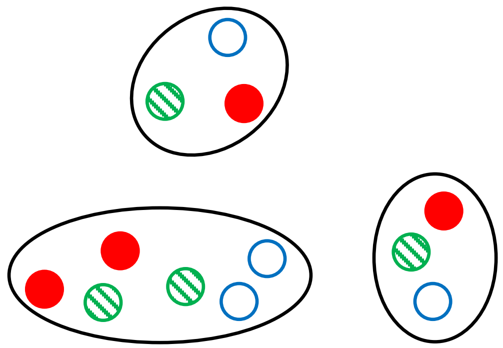
Balanced problem solved	Balance achieved	Approximation factor	Subroutine used	Soubroutine approximation
1-center	1	3	1-center	2
k-center	$1/t'$	4	k-center	2
k-median	1	$2+\sqrt{3}+\epsilon$	*k-median*	$1+\sqrt{3}+\epsilon$
k-median	$1/t'$	$t'+1+\sqrt{3}+\epsilon$	*k-median*	$1+\sqrt{3}+\epsilon$

Hardness: it is NP-hard to optimally find a $1/t'$-balanced k-median clustering.

Fairness and Privacy [RS 18]

General fairness results

- Finds a 12-approximate fairlet decomposition on *any* number of colors
- Implies:
  - 14-approximation for *k*-center
  - 15-approximationfor *k*-supplier



Fair and private clustering

- Privacy: lower bounds on the size of clusters
- Results:
  - 40-approximate private and fair *k*-center
  - 41-approximate private and fair *k*-supplier

Strongly private clustering

- Strong privacy: lower bounds on number of points of a color per cluster
- Results
  - 4-approximate strongly private *k*-center
  - 5-approximate strongly private *k*-supplier

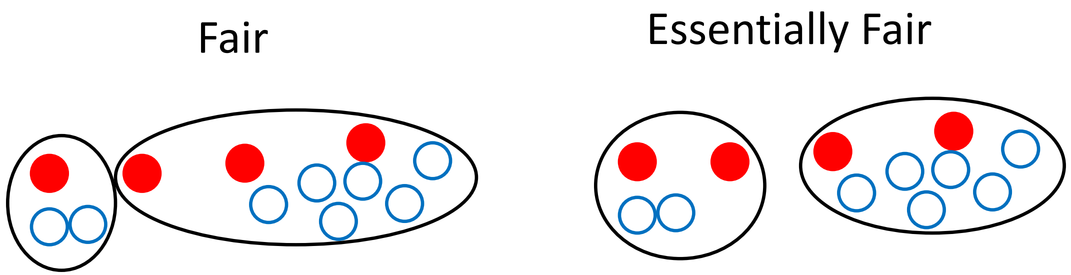
Fairness and Essential Fairness [BGKKRSS 19]

General fairness results

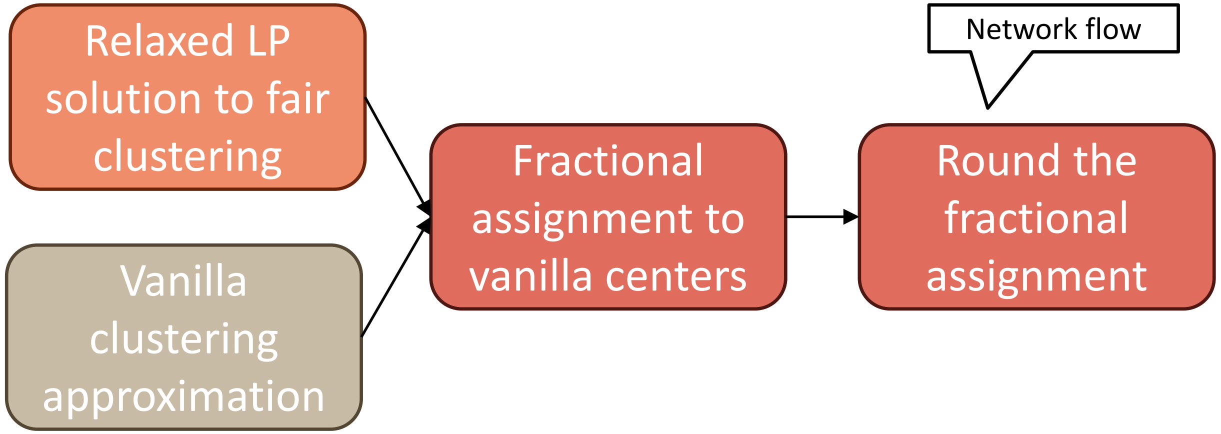
- 5-approximate fair k-center
- 7-approximate fair k-supplier

Essentially fair results

- Clusterings with only *additive* fairness violations:
  - E.g., you can have one extra red point in a cluster
- Results:
  - 3-approximate essentially fair k-center
  - 5-approximate essentially fair k-supplier
  - 3.488-approximate essentially fair facility location
  - 4.675-approximate essentially fair k-median
  - 62.856-approximate essentially fair k-means



4 red, 8 blue: $r/b = 1/2$



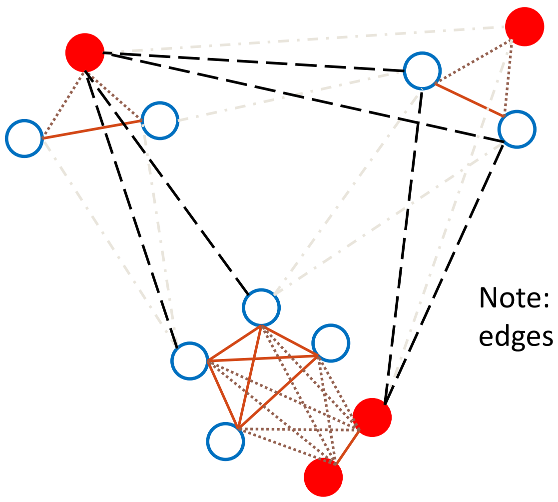
Fair Spectral Clustering [KSAM 19]

Definition – Stochastic Block Model

- There is a fair ground truth clustering
- Generate edges of weight +1 according to color and ground truth cluster

Fair spectral clustering

- Spectral clustering: create a clustering that minimizes the value of RatioCut
  - $RatioCut(S) = \sum_{C \in S} \frac{\sum_{e \in C \times V \setminus C} w(e)}{|C|}$, e.g., the sum of the ratios of the weights exiting a cluster to the size of the cluster
- Proposes a new spectral clustering algorithm:
  - Bounds the error relative to the ground truth clustering
  - Uses $O(n^3)$ time, $O(n^2)$ space



Note: many edges omitted

Same color, same cluster: prob *a*
Same color, different cluster: prob *b*
Same cluster, different color: prob *c*
Different cluster, different color: prob *d*

a > *b* > *c* > *d*

Summary - Balance

First introduced as a concept in 2018 [CKLV 18]

- They also developed the “fairlet decomposition“ technique and came up with initial results

Many of the best approximations are from [BGKKRSS 19], who studied many clustering problems.

Variants explored:

- With privacy [RS 18]
- Spectral Clustering [KSAM 19]
- Essential fairness [BGKKRSS 19]

Demographic Fairness: Bounded Representation

Demographic Fairness – Bounded Representation [BGKKRSS 19]

Instead of requiring perfect balance, we are only constrained by given bounds. There are multiple versions studied:

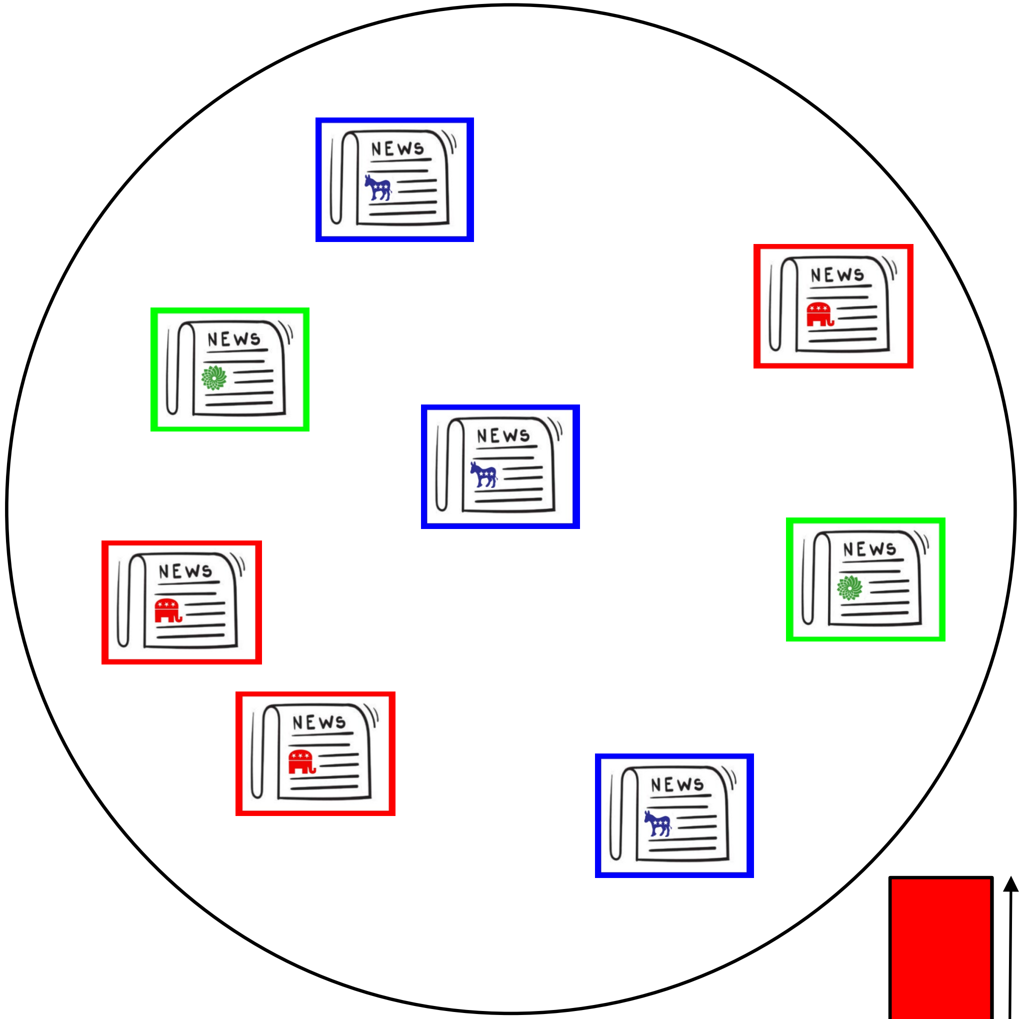
α-bounded for constant α:

- Every color must represent *at most* an α fraction of any cluster

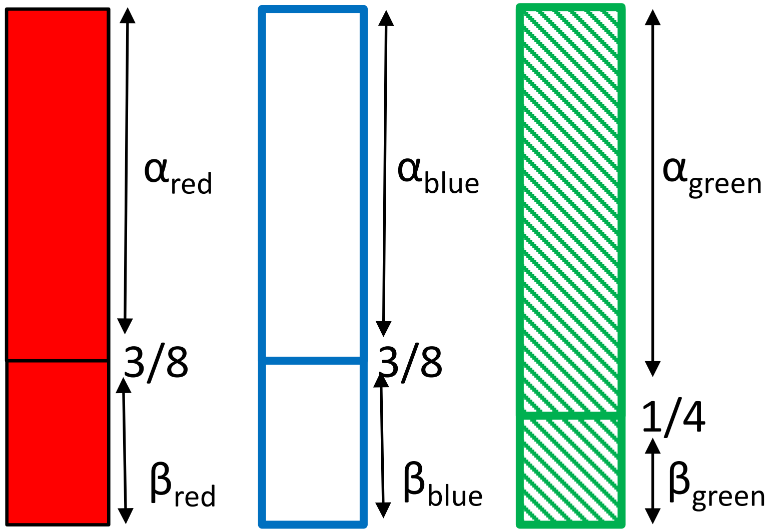
α, β-bounded, for vectors α, β:

- For any color $i \in \{1, \ldots, c\}$, color i must represent *at most* an α_i fraction of any cluster and *at least* a β_i fraction of any cluster

A clustering is fair if every cluster satisfies the bounded representation constraint.



Fair down to α=3/8
Fair down to α_{red}=3/8, α_{blue}=3/8, α_{green}=1/4
Fair up to β_{red}=3/8, β_{blue}=3/8, β_{green}=1/4



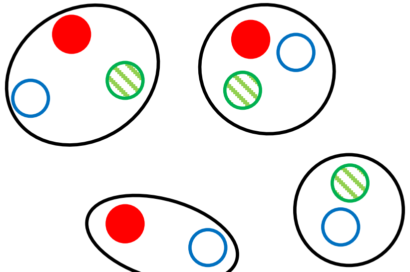
Mitigating Over-Representation [AEKM 19]

Fairness constraint: general upper bound α

Important technique: create a threshold graph on bichromatic edges of weight ≤ τ

Results

- General α: 3-approximation with violation 2
- α=1/t: 3-approximation with violation 1
- α=1/2: 12-approximation with *no* violation



Caplet decomp. on threshold graph (exists)

Merge caplets into a fair clustering

Solve an LP relaxation on weights ≤ 3OPT

Filter facilities: only select distant ones

Round to an integral solution

Here we lose some approximation factor and fairness

Fair Correlation Clustering [AEKM 20]

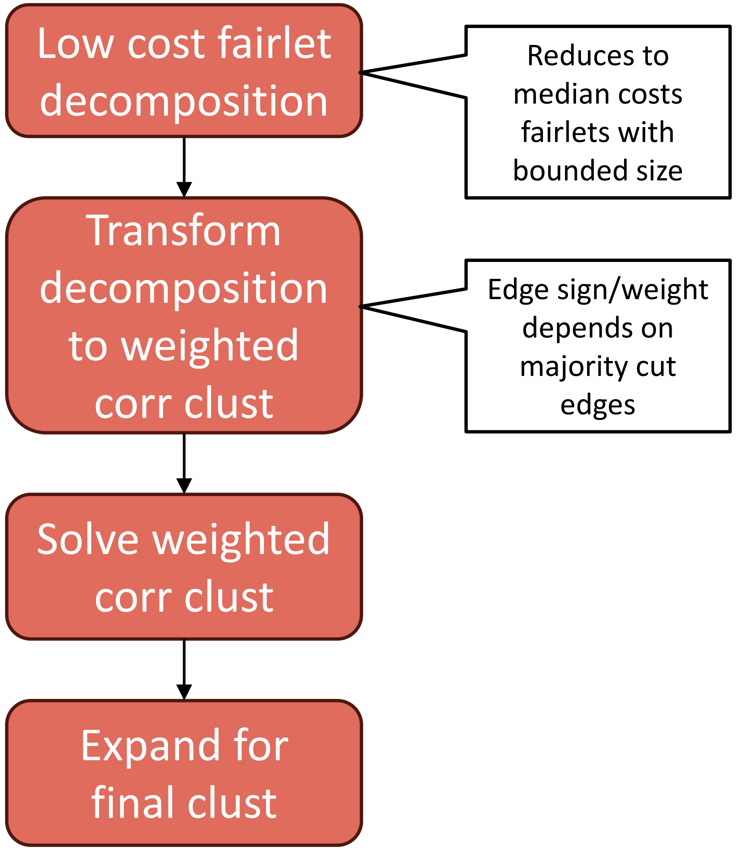
Fairness constraint: general upper bound α (also generalizes further)

Definition – correlation clustering

- Edges are all +1 or -1
- Minimize: the (weighted) sum of the +1 edges between clusters and -1 edges within clusters

Fair correlation clustering (on c colors)

- $\alpha=1/2$: 256-approximation
- $\alpha=1/c$: $(16.48c^2)$-approximation
- $\alpha=1/t$: $O(t\rho)$-approximation for ρ-approximate median cost fairlet decomposition



Fair Hierarchical Clustering [AEKKMMPVW 20]

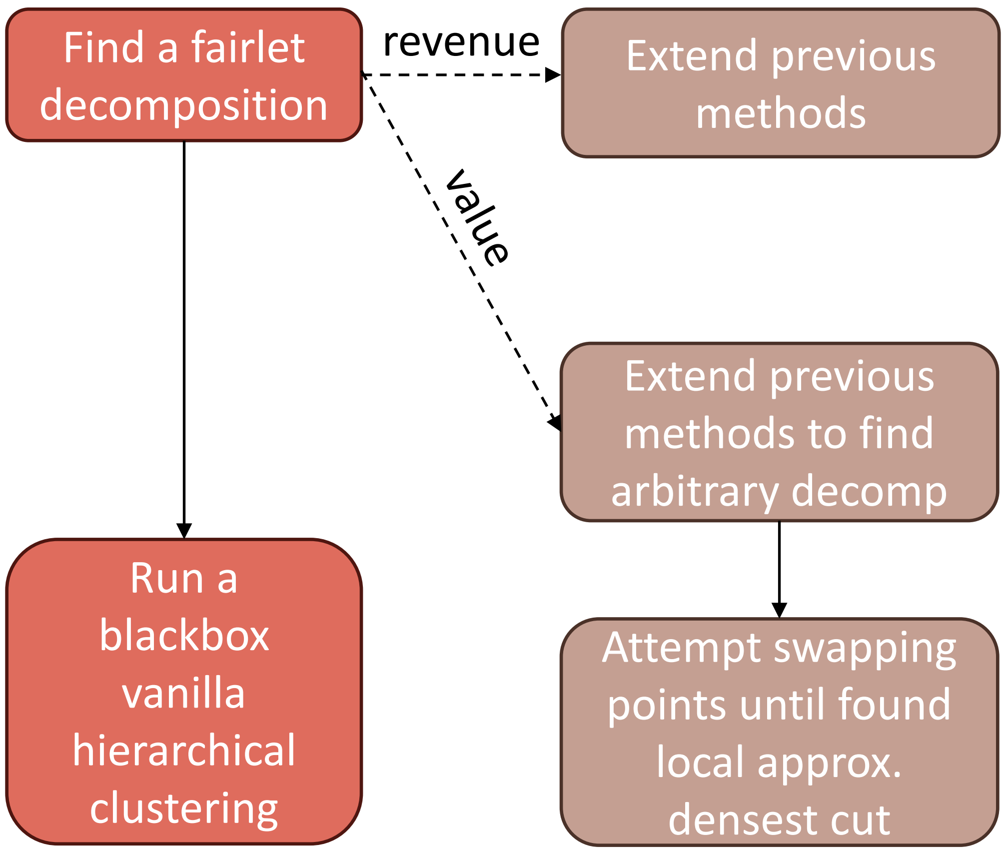
Fairness constraint: general upper bound α (also generalizes further)

Hierarchical clustering objectives

- Cost: initial objective, APX-hard
- Revenue: dual to cost, const-approximable
- Value: cost variant, const-approximable

Results

- Cost: $O(n^{5/6}\log^{5/4} n)$-approximation (highly combinatorial methods)
- Revenue: $(1/3-o(1))$-approximation if α=1/t or 2 colors
- Value: $(2/3-\varepsilon)(1-o(1))$-approximation



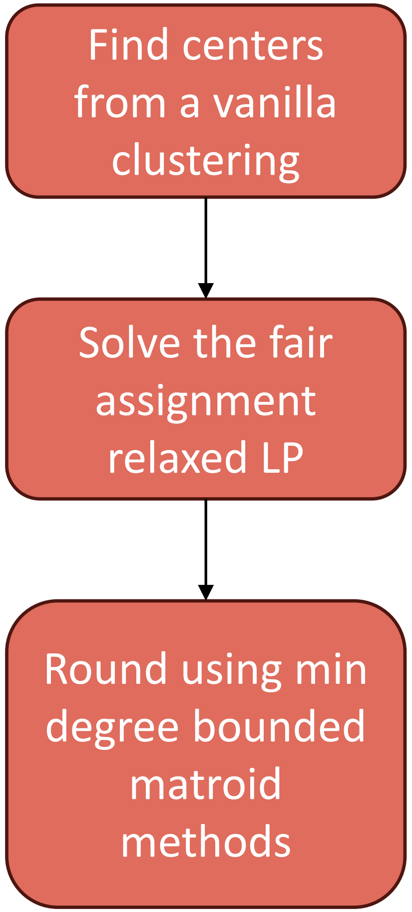
General Bounds, Overlapping Groups [BCFN 19]

Fairness constraint: specific lower and upper bound vectors α, β, also vertices can be in *multiple* groups (bounded by Δ)

Fair assignment problem: given a set of centers, what is the best way to create a fair clustering by assigning points to the centers?

Results

- Given a ρ-approximate vanilla *k*-clustering for the *p*-norm, gives a ρ+2 approximation with additive fairness violation 4Δ+3



Probabilistic Fair Clustering [EBTD 20]

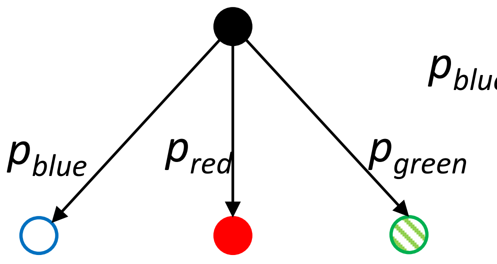
Fairness constraint: specific lower and upper bound vectors α, β

Probabilistic setting

- Colors are not given. Each point has a probability of being assigned some color
- We guarantee that the *expected number* of each color in each cluster is bounded above/below
- Addresses any *p*-norm

Results

- Given a vanilla ρ-approximation, there is a ρ+2-approximation with +1 violation (2 colors)
- Given a vanilla ρ-approximation, there is a ρ+2-approximation with +1 violation (FPT, large clusters)



$p_{blue} + p_{red} + p_{green} = 1$

For every cluster *C*, guarantee:

$$\alpha_i|C| \le E[i(C)] \le \beta_i|C|$$

Frequency of color *i* in cluster *C*

Fixing a Bounded Cost [EBSD 21]

Fairness constraint: specific lower and upper bound vectors α, β

Fair clustering under bounded cost

- Fix an upper bound for the clustering cost
- Minimize the degree of unfairness for any color (i.e., the proportional violation Δ of upper and lower bounds)
  - Utilitarian: minimize the sum of Δs
  - Egalitarian: minimize the maximum Δ
  - Leximin: minimize the maximum Δ, then second largest Δ, …

Results

- Fair clustering (or assignment) under bounded cost is NP-hard
- Given a vanilla approximation, there are approximations for the fair bounded cost problem

Normal linear program

Minimize: $cost(S)$

Subject to: $\alpha_i|C| \leq |i(C)| \leq \beta_i|C|$

New linear program

Minimize: $\sum \Delta$ or $\max \Delta$ or $\max \max \ldots$

Subject to: $cost(S) \leq upper\ bound$

and: $(\alpha_i - \Delta)|C| \leq |i(C)| \leq (\beta_i + \Delta)|C|$

Summary: Bounded Representation

Two versions:

- α-capped clustering
  - Solved with very little to no additive fairness violation [AEKM 19]
- α, β-bounded, with upper and lower bound vectors
  - Solved with additive fairness violation [BCFN 19]

Variants explored:

- α-capped correlation clustering
- α-capped hierarchical clustering
- α, β-bounded probabilistic clustering
- α, β-bounded clustering with bounded cost

However, both are stated in terms of "union closed" constraints

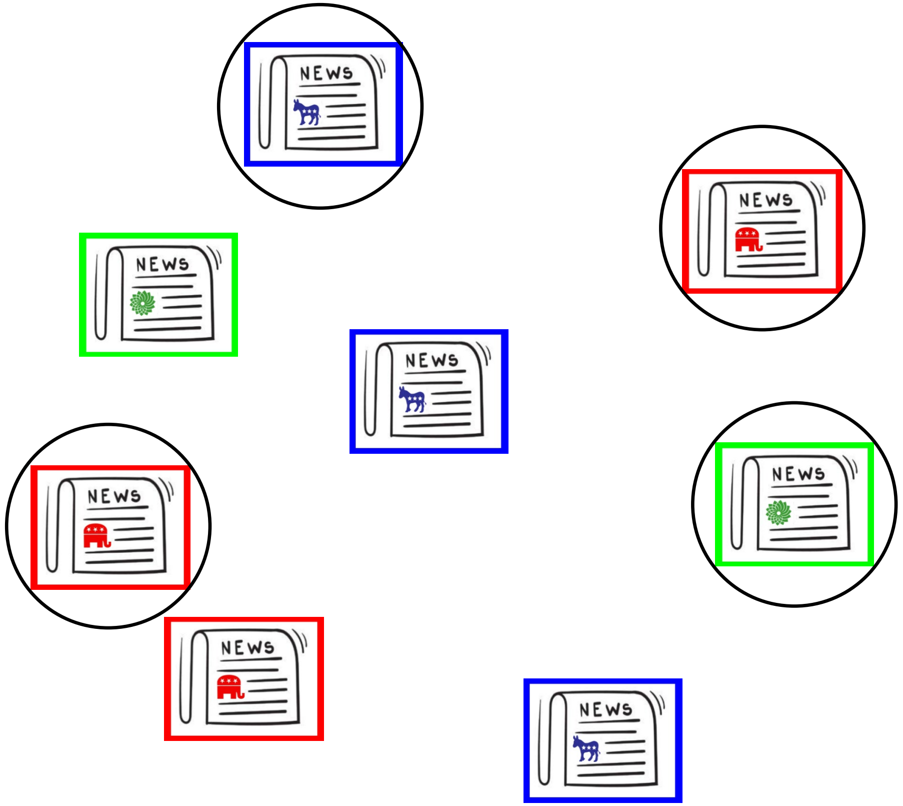
Demographic Fairness: Bounds on Chosen Centers

Demographic Fairness – Bounds on Chosen Centers

Data summarization: do a k-clustering. The resulting centers are then outputted as *representatives* of the data set.

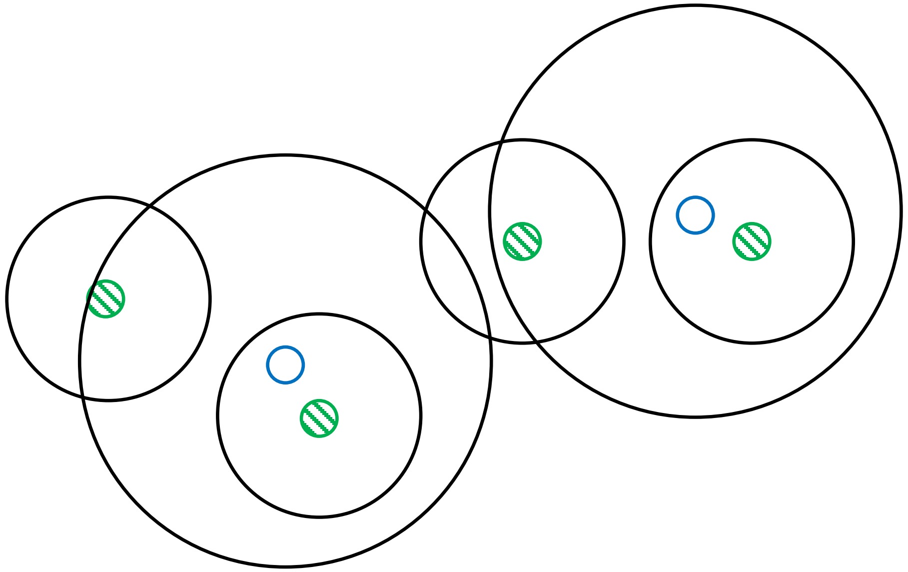
Fairness: for every color i, there must be at least k_i centers of color i.

Let $k_{red} = k_{blue} = k_{\text{green}} = 1$
This clustering is not fair.
However this alternate clustering is fair



Data Summarization [KAM 19, JNN 20]

First introduced the fairness with regards to bounds on chosen centers.



Results [KAM 19]

- Fair data summarization for k-center has a 5-approximation on 2 colors (tight) that runs in time $O(kn)$
- Fair data summarization for k-center has a $(3\times2^{c-1} - 1)$ approximation on c colors that runs in time $O(kc^2n + kc^4)$

Results [JNN 20]

- Fair data summarization for k-center has a 3-approximation on c colors (tight) that runs in time $O(kn)$

Diversity-Aware *k*-Means [TOG 21]

		NP-Hard?	FPT(k)?	Approx factor	Approx method
General case		Yes	No	X	X
c colors	$\sum_{i \in [c]} k_i = k$	Yes	?	8	LP
c colors	$\sum_{i \in [c]} k_i < k$	Yes	?	8	$O(k^{c-1})$ LP calls
2 colors	$k_1 + k_2 = k$	Yes	?	3+ε	Local search
2 colors	$k_1 + k_2 < k$	Yes	?	3+ε	$O(k)$ local search calls

Demographic Fairness: Proportionality

Demographic Fairness – Proportionality

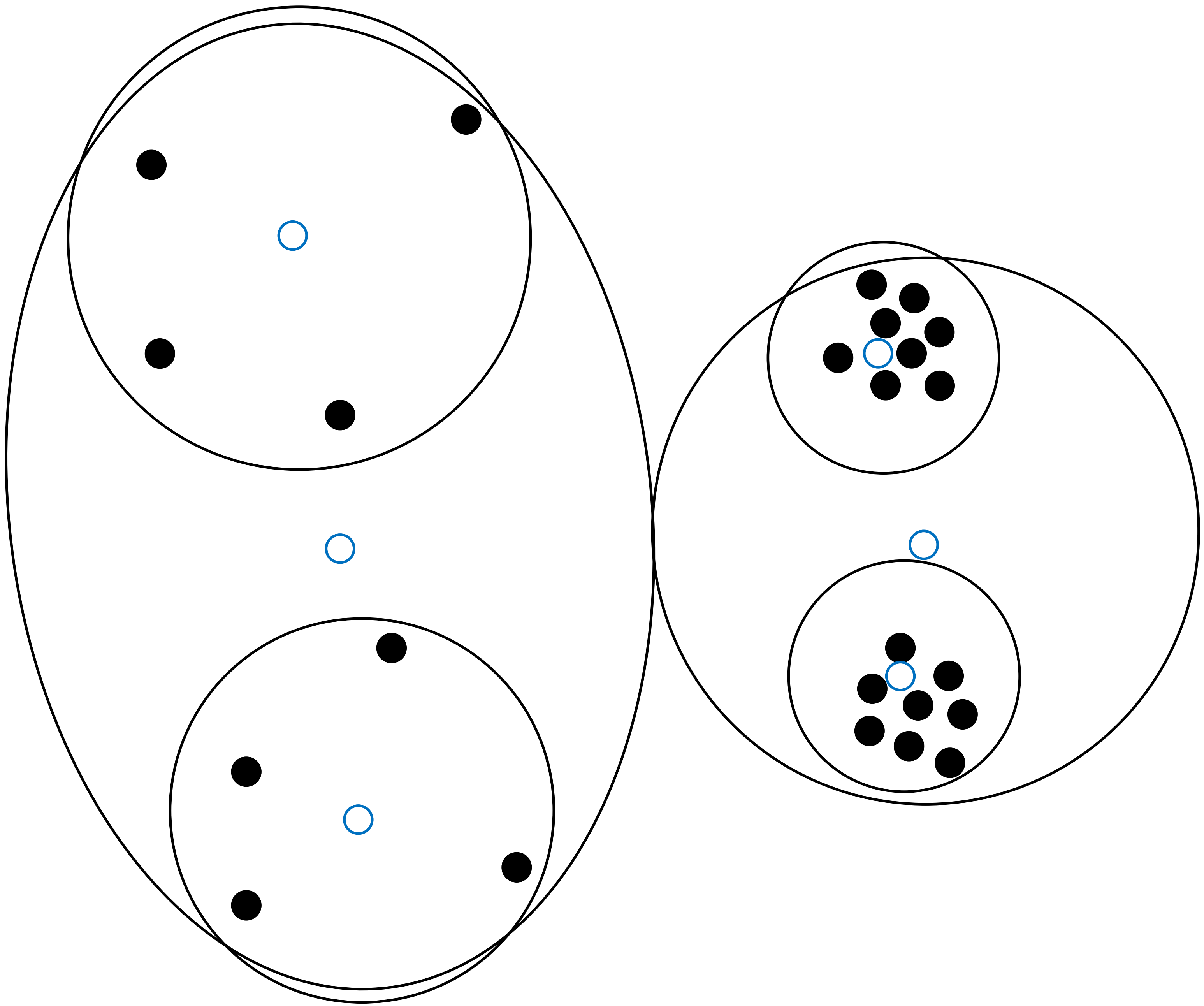
Idea: Every set of at least *n/k* points is entitled to its own cluster.

Blocking coalition: a set of ρ*n/k* points such that we can add a center that is closer to all points in the set than their assigned center.

- “ρ-proportional”

Benefits

- Pareto optimality: Let *X* and *X’* be two proportional solutions. Then there is some point that *X* “treats” at least as well as *X’*.
- Oblivious: Independent of sensitive attributes.
- Robust: Outliers cannot form coalitions.
- Scale invariant

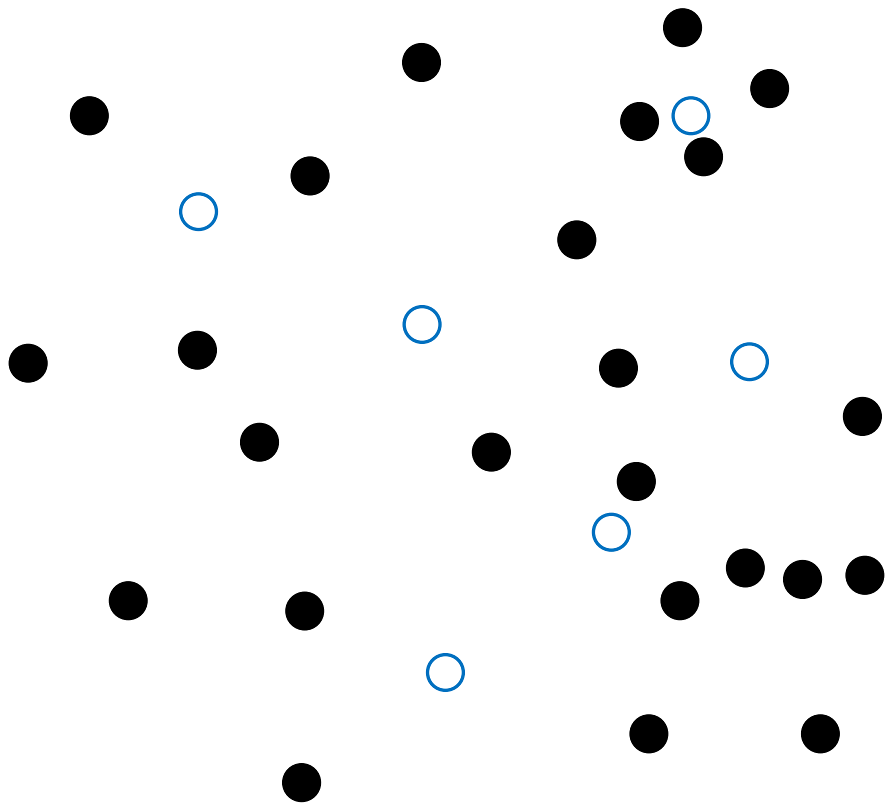


Proportionally Fair Clustering [CFLM 19]

Introduced the problem of proportionally fair clusterings.

Hardness
- There may be no 2-proportional solution

Results
- $(1+\sqrt{2})$-proportional solution
- $O(1)$-proportional solution 8-approximates k-median
- Uniform random sampling approximately preserves the proportionality of any set of centers w.h.p.
- Good heuristic local search algorithm that finds nearly proportional solutions



More Proportionally Fair Clustering [MS 20]

Considers [CFLM 19] in metric spaces defined by different norms.

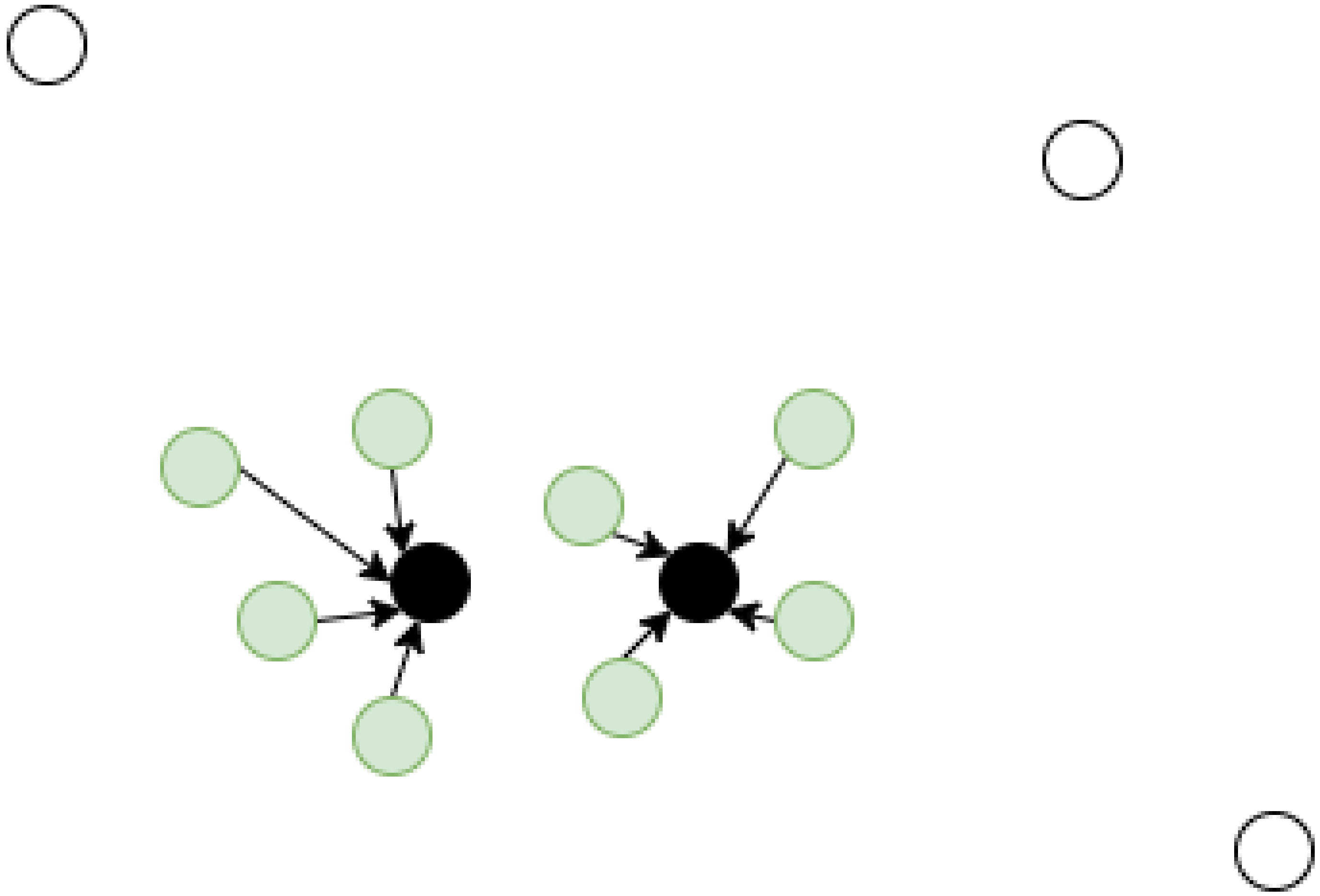
Results

- $(1+\sqrt{2})$ general approximation is really a 2-approximation for L^2
- Shows tightness of $(1+\sqrt{2})$ approximation for L^1 and L^∞
- In L^2, we cannot do better than $2/\sqrt{3}$
- In L^1 and L^∞, we cannot do better than 1.4
- Using tree distance or graph distance when $k \geq n/2$, exact proportionality exists
- In L^2 and many dimensions, checking existence is NP-hard, and the original algorithm is only in NP (it is a PTAS in the dimensionality)
- When there are infinitely many centers, proportionality preservation under random sampling holds even in L^2 and many dimensions

Demographically Fair Clustering with Outliers

k-Clustering with Outliers

- Points $\mathcal{C}$ in metric space with distance $d: \mathcal{C}^2 \rightarrow \mathbb{R}_{\geq 0}$
- Pick $S \subseteq \mathcal{C}$ with $|S| \leq k$
- Pick $\mathcal{A} \subseteq \mathcal{C}$ with $|\mathcal{A}| \geq m$
- Construct $\varphi: \mathcal{A} \rightarrow S$ such that some objective is minimized
- Allowed to exclude a certain number of points from the optimization objective:
  - Robustness: Avoid noise in the data
  - Scarce resources: Servicing only a certain fraction of the population is acceptable



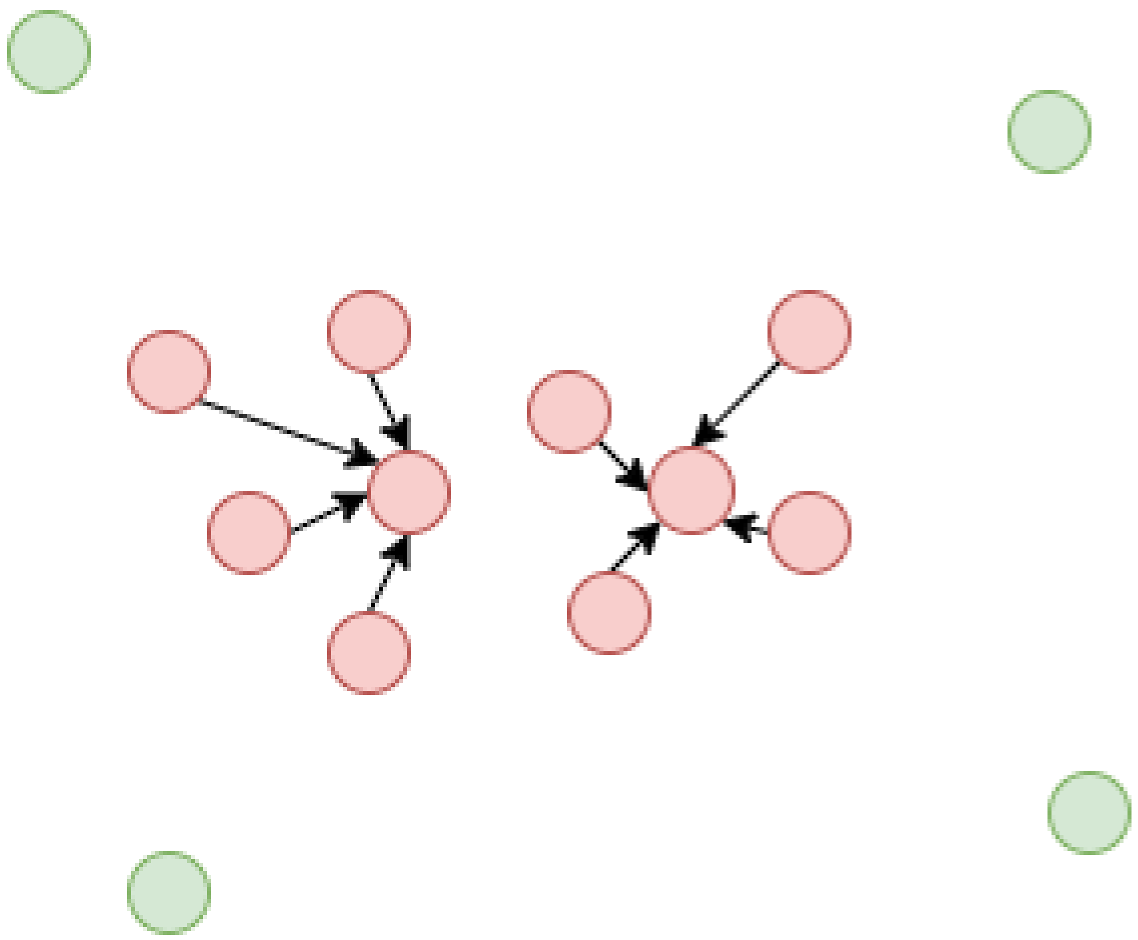
Motivational Examples

- **Clustering Setting:**
  - The points are users of a website.
  - The website wants to cluster its users in groups of high similarity, so that it offers relevant recommendations.
  - The cluster center is thought of as the most representative point of the cluster.
  - Points with unique profiles might be excluded.

- **Facility Location Setting:**
  - Points correspond to cities/towns/counties.
  - A state wants to place vaccination sites in a metric space.
  - Each point should have a vaccination center in close vicinity.
  - Due to scarce resources, it may be acceptable to provide a good covering guarantee to only a fraction of the population.

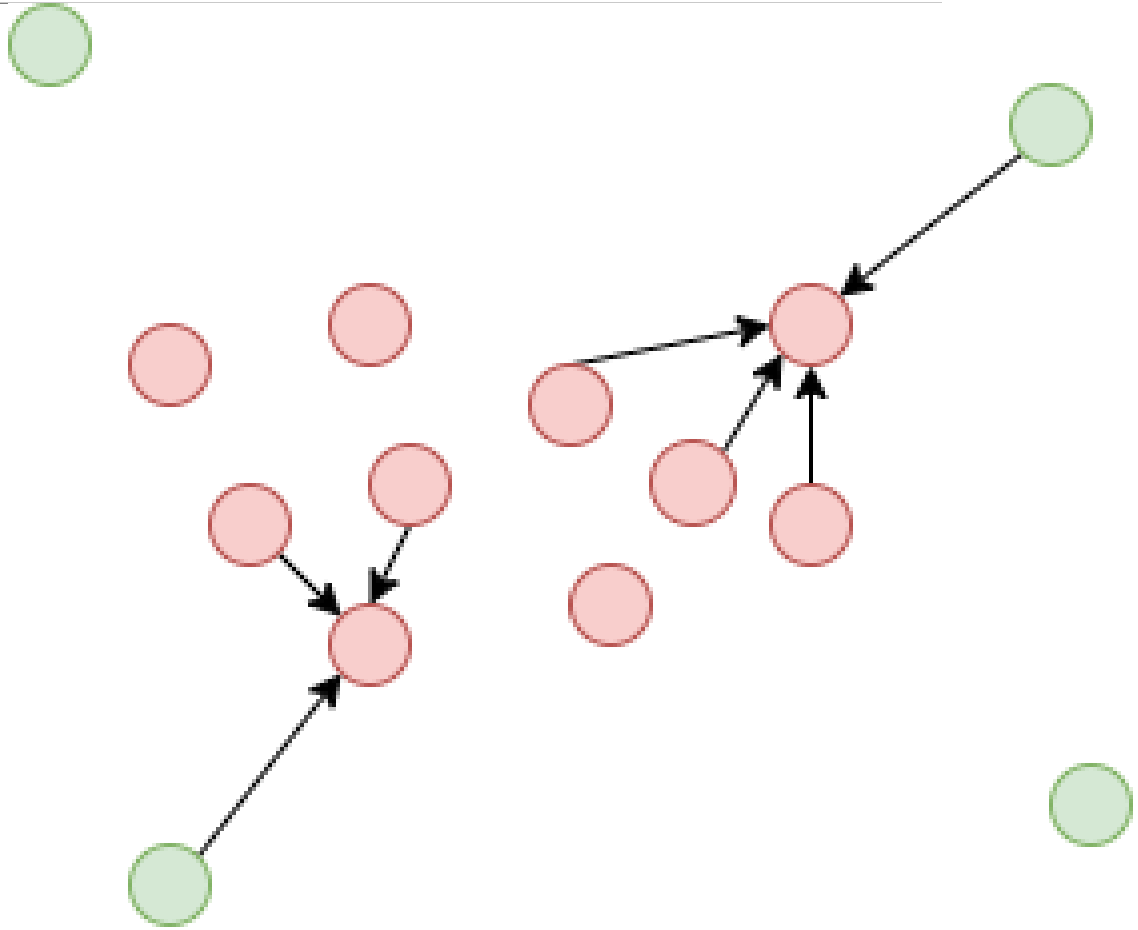
Bias in Clustering with Outliers

- **Being an outlier is disadvantageous!!!**
  - Example 1: Outliers will not receive any recommendations
  - Example 2: Outliers will not enjoy close access to a vaccination center
- Suppose the points of $\mathcal{C}$ come from γ demographic groups $\mathcal{C}_1, \ldots, \mathcal{C}_\gamma$.
- A solution can be biased if it disproportionately views points from certain groups as outliers.



Fair Clustering with Outliers

- Proposed fix:
  - For each group $\mathcal{C}_l$ we are given a value $m_l \geq 0$
  - Instead of $|\mathcal{A}| \geq m$, we now require $|\mathcal{A} \cap \mathcal{C}_l| \geq m_l$ for every $l \in [\gamma]$
- Example with $m_{red} = 5$ and $m_{green} = 2$
- Arbitrary m_l values can capture a plethora of fairness scenarios
  - Equitable treatment, e.g., $m_l \geq |\mathcal{C}_l|/2$ for all $l \in [\gamma]$
  - Preferential treatment, e.g., give a higher coverage guarantee to demographics that really need it



Results

- The problem has only been studied for the k-center objective, i.e., minimize $\max_{j \in \mathcal{C}} d(j, \varphi(j))$, under the name Fair Colorful k-Center
- It was introduced by Bandyapadhyay et al. ("A Constant Approximation for Colorful k-Center"- ESA 2019), who gave a 17-approximation algorithm for it in the Euclidean plane, when $\gamma = O(1)$.
- Anegg et al. ("A Technique for Obtaining True Approximations for k-Center with Covering Constraints") and Jia et al. ("Fair Colorful k-Center Clustering") independently gave a 4-approximation and a 3-approximation respectively, both appearing in IPCO 2020.
- Both of the above algorithms work for general metrics, when $\gamma = O(1)$.
- Anegg et al. also showed that when γ is not a constant, there cannot exist any non-trivial approximation for the problem, unless P=NP.

Socially Fair k-Clustering

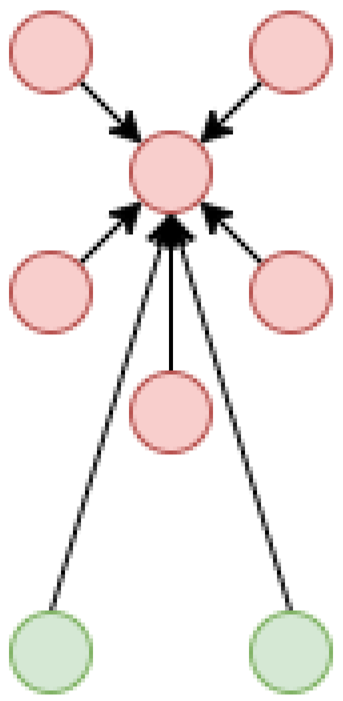
Motivation

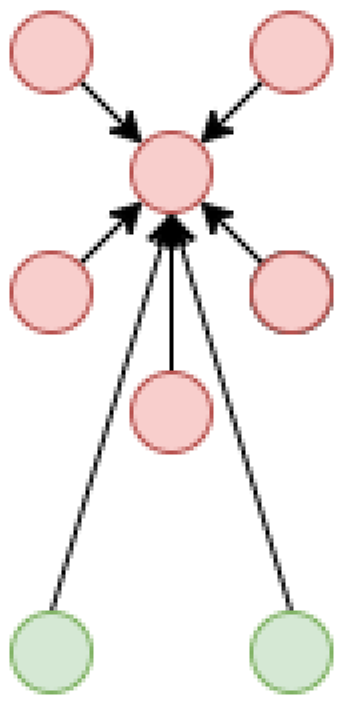
- In many clustering or facility location applications the quantity is $d\big(j, \varphi(j)\big)$ (referred to as "assignment distance") is what really matters.
  - Clustering: It measures how representative $\varphi(j)$ is for j.
  - Facility Location: It represents the distance j needs to travel in order to reach its service provider $\varphi(j)$.

- The smaller $d\big(j, \varphi(j)\big)$ is the more satisfied the point j.
  1) Recall the previously mentioned recommendation system application.
  2) Recall the previously mentioned vaccination sites allocation application.

- Conclusion: If $\mathcal{C}$ consists of γ demographic groups $\mathcal{C}_1, \dots, \mathcal{C}_\gamma$, then we should be fair in terms of the assignment distances provided to the points of different groups.

Example of a Biased Solution





Results

- The problem was introduced independently by Ghadiri et al ("Socially Fair k-Means Clustering") and Abbasi et al. ("Fair Clustering via Equitable Group Representations") at FAccT 2021.
- Both papers demonstrated an $O(\gamma)$-approximation algorithm.
- Makarychev and Vakilian ("Approximation Algorithms for Socially Fair Clustering" – COLT 2021) gave an $O(\frac{log\gamma}{loglog\gamma})$-approximation algorithm. They also showed that this is the best possible approximation ratio for the problem.
- Goyal and Jaiswal ("Tight FPT Approximation for Socially Fair Clustering" – Arxiv 2021) give a tight (3+ε)-approximation algorithm for the problem, that runs in FPT time of $\left(\frac{k}{\varepsilon}\right)^k$.