Individual Fairness in Clustering
High-Level Motivation

- **Demographic Fairness**: Treat each group of points fairly, with respect to how other groups are being treated or with respect to the specific needs of the group at hand.

- **Individual Fairness**: Treat each individual point fairly, with respect to how other points are being treated or with respect to its specific needs.

Does demographic fairness imply individual fairness?
- View each point as a singleton group.
- The concepts of group fairness become vague or ill-defined in this case:
  - Balance: Leads to a single cluster solution
  - Proportionality: Each point is entitled to each its own cluster?
  - Socially fair k-clustering: Reduces to k-center

Demographic fairness cannot adequately capture any individual needs of points.
The Seminal Work of Dwork et al.

- A very important work in the area of Individual Fairness
- Dwork et al. (“Fairness Through Awareness” – ITCS 2012) introduced a groundbreaking concept of individual fairness in the context of classification.

**Similar individuals should be treated similarly**

- It will help us in our taxonomy of individually fair notions for clustering
  1) Definitions that follow the Dwork et al. paradigm
  2) Definitions that diverge from it
Individually-Fair Clustering Models that Follow the Dwork et al. Paradigm
Similar individuals should be treated similarly

Two questions that need to be answered:
1) How can we define similarity in the context of clustering?
2) What constitutes similar treatment in a clustering setting?

The first question is not really important.
The second question is of much more significance.
Similar treatment in terms of same cluster placement

- **Motivational Example:**
  - Suppose a company wants to cluster its employees into k groups
  - People in the first cluster will receive the highest amount of raise, the people in the second cluster the second highest raise, and so on.
  - Suppose that employee X is very similar to employee Y.
  - If Y is placed in a cluster that receives a better amount of raise, then X would arguably feel unfairly treated.

- In such cases, similar points should be placed in the same cluster
Probabilistic Pairwise Fairness – Definition of Similarity

- Introduced by Brubach et al. (“A Pairwise Fair and Community-preserving Approach to k-Center Clustering” – ICML 2020)

- Definition of similarity:
  - For every pair of points $j, j' \in C$ we are given a value $\psi_{j,j'} \in [0,1]$ indicating their true similarity.
    - The smaller $\psi_{j,j'}$ is the more similar the two points.

- The values $\psi$ can be different from the metric $d$:
  1) Encoding of redundant features in $d$
  2) $\psi$ can be the similarity as perceived by the individuals
Probabilistic Pairwise Fairness – Definition of Similar Treatment

- How can we mitigate unfair behavior?
- Avoid situations where two similar points are deterministically separated

**Randomization can imply fairness**

- Seek a randomized solution that separates $j, j'$ with probability at most $\psi_{j,j'}$
  - Choose $S$ with $|S| \leq k$
  - Construct efficiently sampleable distribution $\mathcal{D}$ over assignments $\varphi: \mathcal{C} \to S$ such that
    \[
    \Pr_{\varphi \sim \mathcal{D}}[\varphi(j) \neq \varphi(j')] \leq \psi_{j,j'}
    \]
  - Minimize some metric related objective
Probabilistic Pairwise Fairness - Results

- Brubach et al. ("A Pairwise Fair and Community-preserving Approach to k-Center Clustering" – ICML 2020) introduced the problem and gave a $\log k$-approximation algorithm for the k-center objective.
  - The algorithm works when $\psi_{j,j'} = \left\{ \frac{d(j,j')}{R}, 1 \right\}$, for some $R > 0$.
  - Very efficient algorithm
  - Bounded PoF

- Brubach et al. ("Fairness, Semi-Supervised Learning, and More: A General Framework for Clustering with Stochastic Pairwise Constraints" – AAAI 2021) gave constant factor approximations for all k-center, k-median and k-means
  - The values $\psi_{j,j'}$ are arbitrary
  - Not that efficient – LP based
Distributional Individual Fairness

- Introduced by Anderson et al. ("Distributional Individual Fairness in Clustering" – Arxiv 2020).
- Similarity defined exactly as in Brubach et al. That is with values $\psi_{j,j'}$.
- Pick $S \subseteq \mathcal{C}$ with $|S| \leq k$
- For each $j \in \mathcal{C}$ find distribution $\varphi_j$ over $S$
- Fairness constraint:
  - Metric $D$ measuring statistical proximity
  - $D(\varphi_j, \varphi_{j'}) \leq \psi_{j,j'}$
- Difference with the model of Brubach et al.
  - Brubach et al. return an actual assignment $\varphi: \mathcal{C} \rightarrow S$
  - Brubach et al. upper bound the separation probability
    - Example: For $j, j'$ both $\varphi_j$ and $\varphi_{j'}$ are the uniform distribution over $S$
- Anderson et al. give constant factor approximation algorithms for all k-center, k-median and k-means
Similar Treatment is Terms of the Assignment Distance

- In many applications the quantity \( d(j, \varphi(j)) \) (assignment distance) is what really matters
  - Clustering: It measures how representative \( \varphi(j) \) is for \( j \).
  - Facility Location: It represents the distance \( j \) needs to travel in order to reach its service provider \( \varphi(j) \).

- The smaller \( d(j, \varphi(j)) \) is the more satisfied the point \( j \).

- Suppose \( j' \) is similar to \( j \) and \( d(j', \varphi(j')) \ll d(j, \varphi(j)) \).

  \( j \) is justified to feel unfairly treated
Motivational Example

- The points of $\mathcal{C}$ correspond to users of an e-commerce site.
- $d(j, j')$ measures how similar the profiles of $j$ and $j'$ are.
- The website wants to choose $k$ representative users $S \subseteq \mathcal{C}$ (according to some objective function) and construct an assignment $\varphi: \mathcal{C} \to S$.
- User $j$ will receive recommendations based on $\varphi(j)$’s profile.
- The smaller $d(j, \varphi(j))$ is the more relevant the recommendations $j$ receives.
- If $j$ considers $j'$ as similar to itself, then it perceives $d(j', \varphi(j')) \ll d(j, \varphi(j))$ as unfair treatment.
α-Equitable k-Center

- Introduced by Chakrabarti et al. (“A New Notion of Individually Fair Clustering: α-Equitable k-Center” – AISTATS 2022)

- Every point $j$ has a set of other points $S_j \subseteq C$ that it perceives as similar to itself
  - This is how similarity is modeled in this work
  - Has advantages over the modeling with the $\psi$ values: more easily constructable

- We are also given a value $\alpha \geq 1$.

- Ask for $S \subseteq C (|S| \leq k)$ and assignment $\varphi: C \rightarrow S$ that minimize the k-center objective $\max_{j \in C} d(j, \varphi(j))$.

- **Fairness Constraint:** For every $j \in C$ and $j' \in S_j$ ensure that $d(j, \varphi(j)) \leq \alpha \cdot d(j', \varphi(j'))$
  - The smaller $\alpha$ is the smaller $\frac{d(j, \varphi(j))}{d(j', \varphi(j'))}$ remains
The parameter $\alpha$

- The smaller $\alpha$ is the smaller $\frac{d(j, \varphi(j))}{d(j', \varphi(j'))}$ remains.
  - $\alpha = 4$
  - $\alpha = 1$

- A value of $\alpha$ close to 1 would give the most equitable/fair solution

- For what values of $\alpha$ is the problem well-defined?
  - For $a < 2$ there exist instances that admit no feasible solution
  - For $a \geq 2$ we can always find a feasible solution
The results of Chakrabarti et al.

- A very efficient algorithm that returns a solution of cost $5(R^* + R_m)$
  - $R^*$ is the value of the optimal solution
  - $R_m = \max_{j \in \mathcal{C}, j' \in S_j} d(j, j')$

- When $d$ is a good estimate of similarity: $R_m = O(R^*)$

- Under some mild conditions on the sets $S_j$ the algorithm has bounded PoF
Notions of Individual Fairness in Clustering that do not follow the Dwork et al. paradigm
A Center in my Neighborhood

- Suppose we want to solve a classical $k$-clustering problem on a set of points $\mathcal{C}$
  - Find $S \subseteq \mathcal{C}$ ($|S| \leq k$) and assignment $\varphi : \mathcal{C} \rightarrow S$ such that $\sum_{j \in \mathcal{C}} d(j, \varphi(j))^p$ is minimized

- Even though the global objective function might be minimized, individual points may have different requirements in terms of $d(j, \varphi(j))$
  - Recall the vaccine site allocation example.

- Each $j$ has a value $r_j$, and we should make sure that $d(j, \varphi(j)) \leq r_j$
Results

- Jung et al. (“A Center in Your Neighborhood: Fairness in Facility Location” – FORC 2020) introduced the problem
  - Important result: Even finding a feasible solution to the problem is NP-hard.

- Goal: Find $(\alpha, \beta)$-bicriteria algorithms:
  - $\sum_{j \in C} d(j, \varphi(j))^p \leq \alpha \cdot \text{OPT}$
  - $d(j, \varphi(j)) \leq \beta \cdot r_j$ for every $j$

- A series of papers gave increasingly better results:
  1) Mahabadi and Vakilian (“Individual Fairness for k-Clustering”- ICML 2020). $(O(p), 7)$-bicriteria
     $(2^{1+\frac{\epsilon}{p}}, 8)$-bicriteria
  3) Vakilian and Yalçınler (“Improved Approximation Algorithms for Individually Fair Clustering” – AISTATS 2022) $(16^p, 3)$-bicriteria
Individual Fairness in Clustering with Outliers

- Pick $S \subseteq C$ with $|S| \leq k$
- Pick $\mathcal{A} \subseteq C$ with $|\mathcal{A}| \geq m$ (points to be clustered)

- Being an outlier is disadvantageous!!!
- We have seen how to protect against demographic bias
- What can be interpreted as bias against individuals?

\textbf{Deterministically be chosen as an outlier in every computed solution}
Randomization saves the day: A lottery model for individually fair clustering with outliers

- For each $j \in \mathcal{C}$ we are given a value $p_j \in [0,1]$

- We want a distribution $\mathcal{D}$ over solutions $(S, \mathcal{A})$ such that:
  1) For every $(S, \mathcal{A})$ drawn from $\mathcal{D}$ we have $|S| \leq k$ and $|\mathcal{A}| \geq m$.
  2) $\Pr_{(S,\mathcal{A}) \sim \mathcal{D}}[j \in \mathcal{A}] \geq p_j$ for every $j \in \mathcal{C}$
  3) Some objective is minimized

- We avoid scenarios where certain points are deterministically chosen as outliers

- Through the values $p_j$ we can capture a plethora of fairness concepts:
  - Equitable treatment: $p_j$ is the same for all points
  - Preferential treatment: Points in greater need of service get a higher $p_j$ value
Results

- The problem has only been studied under the k-center objective.

- It was introduced by Harris et al. (“A Lottery Model for Center-Type Problems With Outliers” – APPROX-RANDOM 2017)

- Harris et al. gave a pseudo 2-approximation algorithm.
  - In every solution drawn from $\mathcal{D}$ the coverage guarantee is $(1 - \varepsilon) m$
  - $\Pr_{(S,A) \sim \mathcal{D}}[j \in A] \geq (1 - \varepsilon)p_j$

- Anegg et al. (“A Technique for Obtaining True Approximations for k-Center with Covering Constraints” – IPCO 2020) gave a true 4-approximation algorithm.
Fairness based on average distance to the points in your cluster

- **Motivational Example:**
  - Suppose a company wants to cluster its employees into $k$ groups, based on their performance rating for some specific year.
  - Let’s assume that people in the first cluster will receive the highest amount of raise, the people in the second cluster the second highest raise, and so on.
  - Consider some employee $X$ placed in some cluster $C$. Let $c_x$ be the average distance of $X$ to the rest of the points in $C$.
  - If there exists cluster $W$, with $w_x$ be the average distance of $X$ to the rest of the points in $W$, such that $w_x \leq c_x$, then $X$ would arguably feel unfairly treated.
Formal Definition and Results

- Given a set of points $\mathcal{C}$, partition it into $k$ sets $\mathcal{C}_1, \ldots, \mathcal{C}_k$ such that:
  - For every $i \in [k]$ and each $j \in \mathcal{C}_i$, $\frac{1}{|\mathcal{C}_i| - 1} \sum_{j' \in \mathcal{C}_i} d(j, j') \leq \frac{1}{|\mathcal{C}_{i'}|} \sum_{j' \in \mathcal{C}_{i'}} d(j, j')$ for all $i' \neq i$

- The problem was introduced by Kleindessner et al. (“A Notion of Individual Fairness for Clustering” – Arxiv 2020).

- Main result: For $k \geq 2$, it is NP-hard to decide if such a clustering exists

- When the metric space is the Euclidean line, the problem can be solved efficiently.